Subiectul I

RL2 (0 ∞) – format din multimea semnalelor vectoriale exponential stabile
F(t) = 0 pt t<0
= M e la Nt ξ t>=0

RL2(-∞ 0) – multimea semnalelor vectoriale,cu coeficienti reali , antistabile
F(t)= n e la Nt ξ t<=0
= 0 t>0

RL2(-∞ +∞)= suma lor

RL2+ = Lb(RL2 (0 ∞)) –multimea vectorilor fractiilor rationale cu coeficienti reali in variabila S , fractii stabile strict proprii si fara poli pe axa imaginara.

RL2- =Lb(RL2(-∞ 0))-multimea vectorilor fractiilor rationale cu coeficienti reali in variabila S, fractii stabile strict proprii , antistabile si fara poili pe axa imaginara.

RL2= Lb(RL2(-∞ +∞))-fractii rationale cu coeficienti reali , stabile si antistabile si fara poli pe axa imaginara.

RL∞(RH∞) – multimea matricilor de transfer ce reprezinta aplicatii de la un spatiu RL2 de dimensiune m la un spatiu RL2 de dimensiune p , aceste matrici de transfer au elemente fractii rationale in variabila S cu coeficienti reali , proprii si fara poli pe axa imaginara.

RH∞+ - multimea matricilor de transfer ce reprezinta aplicatii de la un spatiu RL2+ de dimensiune m la un spatiu RL2+ de dimensiune p matrice de transfer cu elemente fractii rationale cu coeficienti reali in variabila S, proprii , stabile , fara poli pe axa imaginara.

RH∞- - multimea matricilor de transfer ce reprezinta aplicatii de la un spatiu RL2- de dimensiune m la un spatiu RL2- de dimensiune p matrice de transfer cu elemente fractii rationale cu coeficienti reali in variabila S, proprii , antistabile , fara poli pe axa imaginara.

RH∞= RH∞+ + RH∞-

Reprez. Structurala in circuit inchis

U1 -> A10,B10,C10,D10 -> y1

X0= $\begin{bmatrix} x \\ xc \end{bmatrix}$ $\left\{ \begin{array}{l} Xu = A10\,Xu + B10\,u1 \\ Y1 = C10\,Xu + D10\,u1 \end{array} \right.$

=>y2-D22u2=C2x + D21u1
Bara Y2=y2-D22u2 =>
Uc=FcXc+GcC2x+GcD21U1+GcD22U2 =>
(I-GcD22)U2=FcXc+Gc(C2x+D21U1)
U2=(I-GcD22)la -1 FcXc + (I-GcD22) la -1 Gc y2 bara
Bara Fc=(I-GcD22) la -1 Fc
Bara Fc=(I-GcD22)la -1 Gc
U2=baca Fc Xc + bara Gc bara Y2

Pct Xc=AcXc+Bc(C2X+D21U1+D22U2)=AcXc+Bc(bara Y2+D22(FcXc+Gc bara y2))=(Ac+BcD22bara Fc)Xc+Bc(I+D22bara Gc)bara y2
Bara Ac=Ac+BcD22 baraFc
Bara Bc=Bc(I+D22 bara Gc)
Pct X=Ax+B1U1+B2(bara Fc+bara Gcbara Y2)=Ax+B2 bara Fc Xc +B1U1+B2bara Gc(C2X+D21U1)=

=(A+B2bara GcC2)X+B2bara Fc Xc + (B1+B2bara GcD21)U1=(A+B2baraGcC2)X+B2bara FcXc+(B1+B2baraGcD21)U1

$$PctX0=\begin{bmatrix} pct\ X \\ bara\ Xc \end{bmatrix}=\begin{bmatrix} A+B2baraGcC2 & B2baraFc \\ baraB2C2 & bara\ Ac \end{bmatrix}$$

$$\begin{bmatrix} X \\ Xc \end{bmatrix} + \begin{bmatrix} B1+B2baraGcD21 \\ bara\ Bc\ D21 \end{bmatrix} U1$$

Y1=C1X+D11U1+D12U2=C1X+ D11U1 + D12baraFcXc + D12baraGc(C1X+D21U1) = (C1+D12Gc C2)X + D12baraFcXc+ (D11+D12baraGcD21)U1

Y1=[C1 + D12GcC2 D12baraFc] $\begin{bmatrix} X \\ Xc \end{bmatrix}$ +

[D11+D12baraGcD21]U1

Semnificatia fizica a normei ∞ pentru o f.d.t a unui system stabil siso reprezinta o masura a energiei semnalului de iesire cand la intrare se aplica un semnal de tip energie unitara.

Semnif. Fizica a normei 2 pt o f.d.t a unui sist. Stabil siso reprez. o masura a energy. Semnalului de iesire cand la intrare se aplica un semnal de tip impuls unitar.

Semnif. Fizica a normei 2.

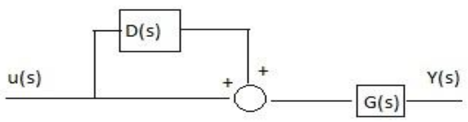
$$\frac{1}{2}\int_{-\infty}^{+\infty} \sum_{i=1}^{k}[\sigma i[H(j\omega)]]d\omega$$

Norma ∞ = sup б[H(jω)]

Norma ∞ a unei f.d.t a unui sist. Stabil reprez. maximul energiei semnalului de iesire dintre toate semnalele de energie unitara care se aplica la intrare.

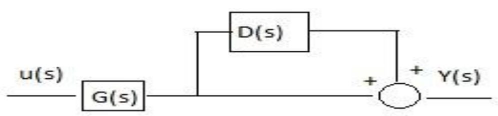
Subiectul II

Incertitudini multiplicative pe intrare



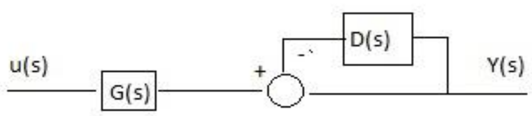
GΔ(s)=(1+Δ(s)$^{-1}$ – siso GΔ(s)=(I+Δ(s))- mimo

Incert. Multiplicativa pe iesire



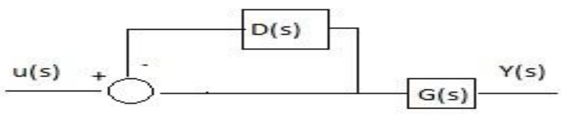
GΔ(s)=(1+Δ(s))G(s)-siso , GΔ(s)=(I+Δ(s))G(s) – mimo

Multiplicativa inversa pe iesire



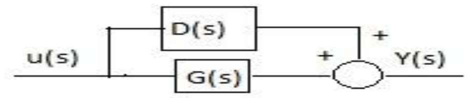
GΔ(s)=$\frac{1}{1+\Delta(s)}G(s)$- siso , GΔ(s)=(I+Δ(s))$^{-1}$G(s)-mimo

Multiplicativa invers pe intrare



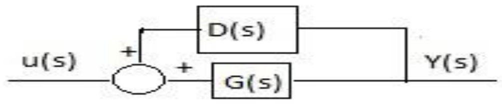
GΔ(s)=G(s)$\frac{1}{1+\Delta(s)}$ $siso$, GΔ(s)=G(s)(I+Δ(s))$^{-1}$-mimo

Aditiva pe iesire



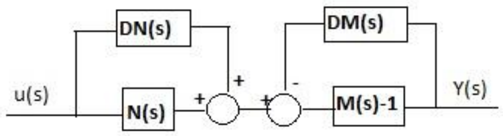
$G\Delta(s)=G(s)+\Delta(s)$-siso, $G\Delta(s)=G(s)+(I+\Delta(s))^{-1}$-mimo

Aditiva pe intrare



$G\Delta(s)=\frac{G(s)}{1+G(s)\Delta(s)}\,siso$, $G\Delta(s)=G(s)(I+G(s)\Delta(s))^{-1}$

In factori coprimi



$G(s)=M(s)N(s)$, $G\Delta(s)=\frac{M(S)}{1+M(s)\Delta M(s)}\,(N(s)+\Delta N(s))$

Robustetea stab. si perform. in cazul siso

Robustetea- satisfice stabilitatea si performanta in conditiile in care conducerea s-ar face asupra unui model real.

$\Delta m(j\omega)\to\xi$ apartine [0 1]

$\xi\to 0$ $L\Delta(j\omega)\to L(j\omega)$

$\xi\to 1$ $L\Delta(j\omega)=G\Delta(j\omega)L(j\omega)$

-beneficiile conexiunii cu reactive negativa :

$L\Delta=L+L\Delta\to T\Delta=T+T\Delta$

$\Delta T=T\Delta-T=\frac{\Delta L}{(1+L\Delta)(1+L)}$

$\frac{\Delta T}{T}=\frac{\Delta L}{L(1+\Delta L)}$sensibilitatea relative in c.i.

Robustetea stabilitatii : (siso)

$|1+L(j\omega)|$ in raport cu (0,j0) sau

$|L(j\omega)|$ in raport cu (-1,j0)

Criteriul Nyquist :-restrans, $|1+L(j\omega|$ nu are poli in C+

-generalizat ……..are poli in C+

$|1+L\Delta(j\omega)|$ -incertitudinea $|\Delta(j\omega) <= lm(\omega)$

-Stabila-> bucla nominala stabila $|1+L(j\omega)$

Satisface criteriul Nyquist restrans si generalizat

$\to|1+(1+\Delta m(j\omega))L(j\omega)|>0$ oricare $\omega>0$ si $|\Delta m(j\omega)|<lm(\omega)$

$|1+(1+\xi\Delta m(j\omega)L(j\omega)|>0$ oricare ξ apartine [0 1] , $\omega>0$

$|1+\frac{\xi\Delta m(j\omega)L(j\omega)}{1+L(j\omega)}|>0\to|\frac{\Delta m(j\omega)L(j\omega)}{1+L(j\omega)}|<1\to$

$|\Delta m(j\omega)|*|T(j\omega)|<1\to|t(j\omega)<\frac{1}{\Delta m(j\omega)}$

$\frac{1}{|\Delta m(j\omega)|}>\frac{1}{lm(\omega)}$ Daca $T(j\omega)<\frac{1}{lm(\omega)}\to$

$|T(j\omega)<\frac{1}{lm(\omega)}$ $lm(\omega)=W3(j\omega)$

[0 ωT]-lungimea de banda in raport cu T

Pt ω apartine BIF |L(jω)|<<1 -> |T(jω)|=|L(jω)|
Compensatorul trebuie proiectat astfel incat ω e BIF
|L(jω)|<=$|\frac{1}{W3(j\omega)}|$ (caracteristica de frecventa sa se gaseasca dedesubtul Z.S in BIF)

Robustetea performantelor (siso) -performantele sunt bine satisfacute in BJF
|ξ(jω)|=|S(jω)||r(jω)|=$|\frac{1}{1+L\Delta(j\omega)}|$|r(jω)|
|1+LΔ(jω)|>=mp(ω) => ω eBJF L(jω)=LΔ(jω)
|1+(1+Δm(jω))L(jω)|>= mp(ω) , ω e[0 ωs] =>
|L(jω)||1+Δm(jω)|>=mp(ω)

$$\frac{mp(\omega)}{|1-lm(\omega)} \geq \frac{mp(\omega)}{|1-\Delta m(j\omega)|} \geq \frac{mp(\omega)}{|1+\Delta m(j\omega)|}$$

|Δm(jω)|<|lm(ω)|

|1+Δm(jω)|>= 1-|Δm(jω)|>=|1-lm(ω)|
Daca |L(jω)|>= $\frac{mp(\omega)}{|1-lm(\omega)}$ => |L(jω)|>= $\frac{mp(\omega)}{|1+\Delta m(j\omega)|}$
mp(ω)=|W1(jω)|
Conditia de performanta robusta impune ca compensatorul sa fie astfel proiectat ca in BJF caracteristica de frecventa sa se gaseasca deasupra Z.P . Robustetea stabil. Si perform. Este asigurata daca se proiecteaza compensatorul a.i. functia caracteristica sa se gaseasca deasupra Z.P in BJF si dedesubtul Z.S in BIF , iar traversarea trebuie sa se faca in BMF.

Robustetea stabil. Si perform in cazul MIMO:
Stabilitatea :
L(s)=G(s)K(s) ; S(s)=$(I+L(s))^{-1}$;
R(s)=K(s)$(I+L(s))^{-1}$=K(s)S(s) ; T(s)=L(s)S(s);
ξ(s)=r(s)-y(s)=r(s)-G(s)K(s)ξ(s) ; r(s)=ξ(s)(I+G(s)K(s);
ξ(s)=S(s)r(s)
u(s)=K(s)ξ(s)=K(s)$(I+L(s))^{-1}$r(s)=R(s)r(s);
y(s)=L(s)$(I+L(s))^{-1}$(r(s)-n(s))+$(I+L(s))^{-1}$ po(s)
y(s)=T(s)(r(s)-n(s))+S(s)po(s)
ξ(s)=$(I+L(s))^{-1}$(r(s)-po(s))+L(s)$(I+L(s))^{-1}$n(s)
ξ(s)=S(s)(r(s)-po(s))+T(s)n(s) ; L(s)=G(s)K(s)
Se calculeaza sensibilitatea buclei in raport cu sensibilitatea relative a caii directe (matricea de transfer a buclei)

S$(I+L)^{-1}$; SΔ=$(I+L\Delta)^{-1}$; T=L$(I+L)^{-1}$;TΔ=LΔ$(I+L\Delta)^{-1}$
ΔL=LΔ-L; ΔT=TΔ-T; Variatia relative in circ. Inchis se obt. Din variatia rel in circ. Deschis diminuata cu factorul $(I+L\Delta)^{-1}$
Criteriul Nyquist –restrans (P+(L(jω))=0) nu inconjoara originea (0 j0)
-generalizat (P+(L(jω))>0) inconjoara originea (0 j0) in sens antiorar

Performanta MIMO :
S+T=I ; S(jω)=$(I+L(j\omega))^{-1}$ |L(jω)|mare => |S(jω)|mic
|T(jω)|=L(jω)$(I+L(j\omega))^{-1}$
T(L(jω))>>1 ; T(I+L(jω))T(L(jω)) ; t(L(jω)<<1
t(L(jω))=t(l(jω))
Ipoteze ale incertitudinilor :
-nu au effect catastrofal
-incertitudinile cresc odata cu frecventa

Subiectul III

a) Procesul acordat p

G(s)= $\frac{2}{s(s+1)}$ y=P(s)*u

$$\begin{bmatrix} y1 \\ y2 \end{bmatrix} = \begin{bmatrix} P11 & P12 \\ P21 & P22 \end{bmatrix} \begin{bmatrix} u1 \\ u2 \end{bmatrix}$$

$$\begin{bmatrix} \begin{bmatrix} y \\ \xi \end{bmatrix} \\ \xi \end{bmatrix} = \begin{bmatrix} P11 & P12 \\ P21 & P22 \end{bmatrix} \begin{bmatrix} r \\ u \end{bmatrix} = \begin{bmatrix} 0 & G \\ 1 & -G \\ 1 & -G \end{bmatrix} \begin{bmatrix} r \\ u \end{bmatrix}$$; y=G*u

ξ=-y+r=r-G*u P11=$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ P12=$\begin{bmatrix} G \\ -G \end{bmatrix}$ P21=[1] P22=[$-G$]

b)

$G = \frac{2}{2(s+1)}$; $y = P(s)*u$; $\begin{bmatrix} y1 \\ y2 \end{bmatrix} = \begin{bmatrix} P11 & P12 \\ P21 & P22 \end{bmatrix} * \begin{bmatrix} u1 = r \\ u2 = u \end{bmatrix}$

$$\begin{bmatrix} \begin{bmatrix} y \\ \varepsilon \end{bmatrix} \\ \varepsilon \end{bmatrix} = \begin{bmatrix} P11 & P12 \\ P21 & P22 \end{bmatrix} * \begin{bmatrix} u1 = r \\ u2 = u \end{bmatrix}$$

y=u*G

ε = r-y = r-u*G

$\begin{bmatrix} \begin{bmatrix} y \\ \varepsilon \end{bmatrix} \\ \varepsilon \end{bmatrix} = \begin{bmatrix} 0 & G \\ 1 & -G \\ 1 & -G \end{bmatrix} * \begin{bmatrix} u1 = r \\ u2 = u \end{bmatrix}$ $P11 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$P12 = \begin{bmatrix} G \\ -G \end{bmatrix}$ P21=1 P22 =G

Ti(P,K)=P11+P12*K*(1-P22*K)$^{-1}$*P21

Ti=$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$+$\begin{bmatrix} G \\ -G \end{bmatrix}$*K*(1-GK)$^{-1}$*1

Ti=$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$+$\begin{bmatrix} \frac{KG}{1-KG} \\ \frac{-KG}{1-KG} \end{bmatrix}$ Ti=$\begin{bmatrix} \frac{KG}{1-KG} \\ 1-\frac{KG}{1-KG} \end{bmatrix}$ = $\begin{bmatrix} T \\ S \end{bmatrix}$

Ts(N,Dm)

$\begin{bmatrix} yd \\ y \end{bmatrix} = \begin{bmatrix} N11 & N12 \\ N21 & N22 \end{bmatrix} * \begin{bmatrix} ud \\ u \end{bmatrix}$; yd=G*u

y=ud+yd=ud+G*u

$\begin{bmatrix} yd \\ y \end{bmatrix} = \begin{bmatrix} 0 & G \\ 1 & G \end{bmatrix} * \begin{bmatrix} ud \\ u \end{bmatrix}$

Ts=N22+N21*Dm(s)(1-N11*Dm(s))$^{-1}$*N12

Ts=G+1*Dm(s)(1-0*Dm(s))$^{-1}$*G = G+Dm(s)*G = (1+Dm(s))*G

c)ωs=1 rad/s ; ωT=100 rad/s

$\frac{1}{W1(s)} = \frac{s+\omega s\xi}{\frac{s}{Ms}+\omega s}$; 2>Ms>1 ;0<ξ<1

W1(s)=$\frac{\frac{1}{1.5}s+1}{s+0.5} = \frac{0.6\,s+1}{s+0.5}$

$\frac{1}{W3(s)}$=$\frac{\xi s+\omega T}{s+\frac{\omega T}{MT}} = \frac{s+\frac{100}{MT}}{\xi s+100}$

W3(s)=$\frac{s+43.4}{0.6\,S+100}$

d) H infinit (h2 e cu h2lqg)

```
numw1=[0.001 0.01 1];
denw1=[1 0.01 0.001];
numw2=[1.1 0.5];
denw2=[1 10];
numw3=[10 200 1000];
```

```
denw3=[1 200 10000];
[aw1,bw1,cw1,dw1]=tf2ss(numw1,denw1);
[aw2,bw2,cw2,dw2]=tf2ss(numw2,denw2);
[aw3,bw3,cw3,dw3]=tf2ss(numw3,denw3);

nump=[1];
denp=[0.0025 0.05 1];
[ap,bp,cp,dp]=tf2ss(nump,denp);
[A,B1,B2,C1,C2,D11,D12,D21,D22]=augment([ap bp;cp dp],[aw1 bw1;cw1
dw1],[aw2 bw2;cw2 dw2],[aw3 bw3;cw3 dw3],[2 2 1 2]);
syspack=mksys(A,B1,B2,C1,C2,D11,D12,D21,D22,'tss');
[contr,Ty1u1]=hinf(syspack);
[ac,bc,cc,dc]=branch(contr);
[ay1u1,by1u1,cy1u1,dy1u1]=branch(Ty1u1);

[ad,bd,cd,dd]=series(ap,bp,cp,dp,ac,bc,cc,dc);
[numd,dend]=ss2tf(ad,bd,cd,dd);
[numdy1u1,deny1u1]=ss2tf(ay1u1,by1u1,cy1u1,dy1u1);
omega=logspace(-2,3,150);
sysd=tf(numd,dend);
sysw1=tf(numw1,denw1);
sysw2=tf(numw2,denw2);
sysw3=tf(numw3,denw3);

[numk,denk]=ss2tf(ac,bc,cc,dc);
Komp=tf(numk,denk);
dltm=tf([2 80],[1 100]);
S=1/(1+sysd);
R=Komp/(1+sysd);
T=sysd/(1+sysd);
figure(1);
bode(sysd,omega);
grid on;
hold on;
bode(sysw1,omega);
bode(dltm,omega);
bode(sysw3,omega);

bode(1/sysw3,omega);
bode(S,omega);
bode(T,omega);
bode(1/sysw1,omega);
figure(2);
sigma(ay1u1,by1u1,cy1u1,dy1u1,omega);
grid;
figure(3);
bode(1/sysw2,R,omega);
grid;
```